Visualizing Geometric Uncertainty of Surface Interpolants

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Abstract

Evaluating and comparing the quality of surface interpolants is an important problem in computer graphics, computer aided geometric design and scientific visualization. *Geometric uncertainty* is a measure of interpolation error, level of confidence or quality of an interpolant that depends upon geometric characteristics of interpolants such as position, normals, isophotes, principal curvatures and directions, and mean and Gaussian curvatures. We present several new techniques for visualizing geometric uncertainty of surface interpolants, that combine the strengths of traditional techniques such as pseudo-coloring, differencing, overlay, and transparency with new glyph and texture-based techniques. The viewer can control an interactive query-driven toolbox to create a wide variety of graphics that allow probing of geometric information in useful and convenient ways. We demonstrate the effectiveness of these techniques by visualizing geometric uncertainty of surfaces obtained by different interpolation techniques – bilinear, C^0 linear, C^2 bicubic B-spline, multiquadrics, inverse multiquadrics and thin plate splines.

Keywords: comparison, geometry, glyphs, interactive, interpolation, probes, surfaces, texture, uncertainty, visualization.

1 Introduction

Central to the work of scientists, engineers and designers is the task of constructing models of data sets obtained by instruments or created by users. Data interpolation is an important example of this task. However, in most situations, there is no clear choice of one model over another. Therefore, scientists, engineers and designers are very keenly interested in comparing the results from different models, and analyzing their relative advantages and disadvantages.

Geometric uncertainty is a measure of interpolation error, level of confidence or quality of an interpolant, that depends on geometric characteristics of interpolants such as position, normals, isophotes, principal curvatures and directions, and mean and Gaussian curvatures. These and other measures of geometric uncertainty will be discussed in Section 2.1.

Visualizing geometric uncertainty is a very valuable aid in evaluating the effectiveness of an interpolation scheme. Although some techniques such as side-by-side display, differencing and pseudo-coloring have been found to be successful to some extent, no one technique is flexible or powerful enough to provide the wide range of information that a user typically seeks. Moreover, most of the past methods provide no control to the user for probing the quality or geometry of the interpolants.

In this work we present new techniques for visualizing geometric uncertainty of surface interpolants, that combine the advantages of traditional techniques such as pseudo-coloring, differencing, overlay and animation with new glyph and texture based techniques. Our system also provides an interactive control to the user for probing geometric information of surface interpolants in many useful and convenient ways.

We demonstrate the effectiveness of these techniques by visualizing geometric uncertainty of surfaces obtained by different interpolation techniques that include multiquadrics, inverse multiquadrics, thin plate splines, bilinear, C^0 linear and C^2 bicubic B-spline interpolation schemes.

2 Background

In this section we describe the previous work on defining and visualizing uncertainty with an emphasis on geometric uncertainty.

2.1 Uncertainty

Uncertainty is a term that has been used to describe several different features of scientific data including error, accuracy, confidence level and quality of data. Error can be defined as the discrepancy between a given value and its true value [GBW94]. Inaccuracy is the difference between the given value and its modeled or simulated value [GBW94]. Confidence level is the level of confidence that can be associated with data and can be computed based on statistical methods or evaluation by scientific judgement [TK93]. Data quality is a very broad term that encompasses many concepts including data validity and data lineage [BBC91].

Geometric uncertainty, likewise, is a scalar or a vector-valued function that captures error, accuracy, quality or confidence level of the geometry of a surface. The geometric characteristics of interest typically include several pieces of geometric information that are based on positional, first, second and sometimes even third derivative information. The first derivative information of interest at a point on the surface includes tangent plane information, normals and isophotes. Given a normal $\vec{N}(p)$ at the point p on a surface and a direction $\vec{L}$ of the light source, the *isophote* surface $I_{\vec{L}}(p)$ can be defined as $I_{\vec{L}}(p) = \vec{N}(p) \cdot \vec{L}$, where $\cdot$ denotes the dot product. There is a continuum of isophote surfaces depending upon the direction of the light source. Contours of isophote surfaces have been used to interrogate surface geometry [HHS+92]. Most of the geometric measures that capture second derivative information are based on minimum and maximum principal curvatures κ_1 and κ_2 and the associated principal directions $\vec{e_1}$ and $\vec{e_2}$ respectively. We refer the reader to any standard textbook on differential geometry for details [dC76]. Important geometric measures for surfaces are Gaussian curvature $K = \kappa_1\kappa_2$ and mean curvature $H = \frac{1}{2}(\kappa_1+\kappa_2)$. Both Gaussian and mean curvatures are geometric invariants that capture the local geometry of the surface. The quantity $\kappa_1^2+\kappa_2^2$ measures the strain energy of flexure and torsion in a thin rectangular elastic plate with small deflection, and is typically used as a standard fairness criterion for surfaces in engineering [HS91]. Third derivative information is captured by the sum of the variations of the principal curvatures along the principal directions, that is, $(\frac{d\kappa_1}{d\vec{e_1}})^2 + (\frac{d\kappa_2}{d\vec{e_2}})^2$, which has also been used as a fairness metric [MS94]. Other more sophisticated criteria have also been adopted [MS94]. In addition, reflection lines, orthotomics and focal surfaces have also been proposed for surface interrogation [HHS+92]. In principal, any function of the above measures or weighted combination of these measures such as the differences between these measures can be used as an estimate of geometric uncertainty. The exact choice depends upon the application at hand.

2.2 Visualizing Uncertainty

Popular techniques for visually comparing surface interpolants are side-by-side comparisons, difference comparison and pseudo-coloring. Franke compared visual aspects of several interpolants by drawing wireframe perspective plots side-by-side [Fra82]. Isophotes have been compared by drawing the contours of isophote surfaces side-by-side [HHS+92]. Difference comparison is a technique where the difference between two images, surfaces or volumes is computed point-by-point and the difference image, surface or volume is rendered. Examples of this occur in comparing images by Tvedt [Tve91]. Pseudo-coloring has been used to compare Gaussian curvature of surface interpolants by Lounsbery et al. [LMD92].

Other techniques for visual comparisons include transparency, overlay and animation. Use of transparency for comparing surface interpolants is presented in [PFN94]. Related concepts of blends (including techniques based on percentage classification of materials), fuzziness, fog or blurs have been proposed in [FLN90, BBC91]. The idea of overlaying two curves or surfaces and connecting the respective points by straight lines or overlaying contour plots is also quite popular. Animation has been used to visualize fuzzy data [Ger92].

Although glyphs or textures have not been used for comparing or visualizing surface interpolants, they are quite common in data displays. Glyphs are symbols that represent data through visual properties such as size, shape, color, position and orientation. They have also been called probes, geometrical primitives, stars, boxes and icons [PG88]. Line segment glyphs have been used in porcupine-type displays of surface normals. Glyphs have been used to represent univariate data [Tuk84, Tuf83]. Different types of glyphs such as stars, Chernoff faces, boxes, profiles, Kleiner-Hartigan tress and Andrew's plots have been used to represent multivariate data [CBB91]. Glyphs for representing vector and tensor fields are shown in [dLvW93]. Texture mapping has been used in scientific visualization [vW91]. Displacement mapping and bump mapping are also standard techniques in computer graphics.

In addition to the techniques mentioned above, most of the work in visualization of uncertainty has been in the field of Geographic Information Systems, for which we refer the reader to [GBW94] or [WSF+95]. We also mention that several techniques have been proposed for visualizing surfaces over surfaces and multi-valued volumetric visualization [FL91, Nie87], but none of them seems to have addressed the question of visually *comparing* surfaces or visualizing geometric uncertainty.

3 Features of the System

The geometric uncertainty visualization system that we have developed is written in C and GL library for SGI platforms using FORMS interface. Currently the system has the capability to read data sets specified on a rectangular grid. We now present an overview of our system for visualizing geometric uncertainty of surface interpolants and the key factors that influenced the design of the system. First, although traditional visualization techniques such as pseudo-coloring or differencing have been successful to some extent, no one technique is flexible or powerful enough to provide the wide range of information that a user typically seeks. Therefore, our system creates a wide range of visualization possibilities that incorporate the complementary advantages of different visualization techniques. Second, in our visualizations, we have attempted to incorporate the important principles of data-ink maximization [Tuf83] and maximum impact [Tuk84] by providing a clutter-free presentation and focusing on the substance of the presentation. More importantly, we are guided by the principle of maximum utility to the user. Therefore, the user is provided with an interactive query-driven toolbox that allows the facility to control many parameters such as geometric uncertainty parameters, subregion selection, scaling, lighting, zooming, translation, rotation, color ramps to create their own views. Moreover, in our visualizations, we have included many retinal or visual variables such as shape, size, and color based on Bertin's classification [Ber83]. We now discuss both these features in greater detail.

3.1 Visualization Techniques

In order to capture diverse geometric information together, we have created visualizations based on *geometry glyphs*. Geometry glyphs are visual objects that convey geometry through its visual properties such as size, shape, color and position. The user can choose between many different shapes that include boxes, spheres and ellipsoids. Shapes, sizes and colors can be mapped to user-preferred geometric parameters. These choices provide a wide range of possible glyphs. We now describe specific examples of some glyphs that we have found useful. A *displacement glyph* (Figure 4) at a point is a thick line or a cylinder or an ellipse or a box, the height of which encodes the geometric information of interest at that point. A *cross-hair glyph* (Figures 6 and 7) consists of two orthogonal planes, the heights of which encode uncertainty of mean and Gaussian curvatures. A *triangular glyph* (Figures 7 and 8) is a vector-glyph that displays the triangular region between two vectors at the same point. We have used triangular glyphs to display the geometric uncertainty of normals and principal curvature directions at a point. We have also created a *volume-filling glyph* (Figure 1) that encloses the volume between two surfaces by spheres whose radii are proportional to the difference between two surfaces.

In order to create visualizations that are clutter-free and easy to perceive, we have used texture mapping for capturing geometric uncertainty information. Three different techniques of texture mapping have been implemented and investigated: displacement mapping, bump mapping and spot mapping. In displacement mapping (Figure 3), one of the surfaces is randomly perturbed in proportion to the geometric uncertainty parameter. In bump mapping, the normals to the surfaces are perturbed. In spot mapping, regions of high relative differences appear spotted (Figure 8). The spot texture or jitter created in the surface highlights the regions of interest without extra gadgets as with glyphs.

Our visualization system also incorporates most of the traditional visualization techniques including side-by-side comparisons, pseudo-coloring (Figures 2 and 5), differencing (Figure 5), overlay (Figure 4), animation and transparency (Figure 7) for visualizing any one geometric feature of interest. The system also allows the user to choose from a wide variety of geometric uncertainty parameters, described in Section 2.1.

Combinations of these techniques provide a richer and more useful class of techniques. For example, Figure 4 combines displacement glyphs with overlay surfaces; Figure 5 combines differencing with pseudo-coloring; Figure 7 combines transparency with cross-hair glyphs and triangular glyphs; and Figure 8 combines spot texture mapping with triangular glyphs. By combining these techniques judiciously, we have created a wide range of new possibilities for probing the geometry of surfaces. Advantages of combining these visualization techniques are presented in Section 3.2.

3.2 Interactive Features

This visualization system provides the user with query-driven interactive control of several features in order to create graphics that are useful and convenient to view. The user is presented with an interactive interface that consists of several menus, sliders, and buttons. Using menus, the user can click on one of the many surface interpolation schemes that are available or the user can choose one of the many visualization techniques such as pseudo-coloring, transparency, and glyphs by a simply click. The visual parameters available with any visualization technique can be interactively changed through a slider. The query-based interaction and region selection is provided through additional interactive windows that pop up through a simple click and provide the user with the option of specifying the parameters by clicking on domain points or by entering the exact coordinates of the domain points.

The system allows standard geometric and viewing transformations such as translation, scaling, rotation and zooming. A 3D-trackball allows user to pick a direction of the light source interactively in order to create an isophote surface.

Visual Parameter Selection: With every visualization technique, there are several visual parameters that can be controlled by the user. In glyph-based techniques the user can choose the display resolution as well as the size, shape and color of the glyphs. In texture-based techniques, the user can choose the randomness factor. In transparency or pseudo-coloring, the amount of transparency or the choice of the color ramp is up to the user. In addition, there are several visual parameters that are not tied to any particular visualization technique. For example, the user can position the lights, choose the intensity and colors of the light and choose material properties of the surface such as the coefficients of reflectivity for ambient, diffuse and spectral light. The user also has the ability to view a wireframe representation or a shaded representation. This flexibility can be used for three different purposes:

1. *To create views that are easy to navigate and understand*: This objective is achieved by mapping visual parameters according to convenience of viewing. For example, the display resolution can be chosen for a dense (Figure 6) or a sparse presentation (Figure 7). Size of the glyphs can be increased if the original glyphs are too small to view indicating that the absolute differences between the two interpolants are very small. A green-red ramp is chosen in Figures 2 and 5 over a standard grey ramp, because it indicates not only the magnitude of the differences between the two surfaces by the brightness, but also the sign of the differences by the color. The amount of transparency has been manipulated in Figure 7 to display a transparent surface where the differences are small and relatively opaque where the differences are large. The randomness factor in displacement mapping has been chosen in Figure 3 to present a certain level of contrast that is meant to represent the level of confidence in the interpolant. Regions of low level of confidence appear uncertain due to its rough texture.

2. *To overload an image with additional cues:* Visual parameters are mapped to the same geometric information in order to reinforce the data. Figure 4 displays two isophotes corresponding to two different interpolants. The differences between the two isophotes are then filled in by displacement glyphs. Both the mappings – overlay and the displacement glyphs – encode the same information about the position of the isophotes. However displacement glyphs provide additional cues. As another example, Figure 2 displays a surface that has been pseudo-colored according to the difference between the two interpolants in addition to the glyphs that encode the same information through their heights. Both the mappings – the pseudo-color and the glyphs – provide the same information but reinforce each other in a strong way to provide a much better understanding of both relative and absolute values.

3. *To create a single graphic that brings together diverse geometric information together:* In order to achieve this objective, visual parameters such as glyph parameters, texture parameters, amount of transparency or the color ramp are mapped to different geometric uncertainty parameters. Figure 7 displays the multiquadric interpolant, where differences between the multiquadric and the thin plate spline interpolant are highlighted using transparency technique, differences in normals are shown by triangular strips and cross-hair glyphs have been utilized to display the differences in mean and Gaussian curvatures. This graphic combines the positional, the first derivative and the second derivative uncertainty information in a single graphic.

Query-Based: This refers to the ability of the user to highlight or display only a part of the entire graphic that satisfies certain constraints or queries. These queries are tied to the geometric properties of the surface. An example of such a query is to display only those glyphs that represent large differences between normals (Figure 8). This facility is important in several situations. An example is when small differences may clutter the presentation and the viewer may want to remove them. Another example is when large differences dominate in a pseudo-colored view and the user wants to remove them in order to focus on regions with intermediate or low values.

Region Selection: This refers to the ability of the user to select certain subregions of interest. For example, the viewer can choose to view only the region around a hill or a saddle point. Our system provides the facility to the user for viewing only that part of graphics that are associated with a curve or a point. The user can select these subregions either by clicking with a mouse or by providing the location of the point or the equation of the curve. This feature is useful for probing the surface at a given point, surrounding regions or along boundary curves. Glyphs along the curves can be animated with *animated probes*. In this case a glyph such as an ellipsoidal ball or a box moves along a curve on one surface and expands or shrinks according to the difference between two surfaces along that curve. The user can control the speed of the probe. Alternatively, the glyphs along the curves can be swept along a desired curve and retained for subsequent viewing in *swept probes* (Figure 2).

4 Implementation and Analysis

We now describe the interpolation schemes and data sets used in the experimentation of our visualization system. We then discuss the results of our experiments.

4.1 Interpolants

We have implemented several interpolation techniques, that are quite popular in computer graphics, computer aided geometric design and scientific visualization applications. These interpolants are C^0 piecewise linear interpolant (based on a triangulation of the data), bilinear interpolant, and C^2 bicubic B-spline interpolant for gridded data. For the bicubic B-spline interpolants, we have used the generalization of not-a-knot boundary condition [Wol90] for constructing tensor-product interpolants. We have also implemented Hardy's multiquadrics, inverse multiquadrics, and thin plate splines. The motivation for choosing these radial interpolants is that these three radial interpolants are the only ones (besides one more radial interpolant) that received an 'A' rating in visual category in Franke's survey [Fra82].

4.2 Examples and Data Sets

We have experimented with Franke's six analytic test functions [Fra82], which include a wide variety of shapes including hills, valleys, cliffs, saddles and a part of a sphere. The equations for these functions are available in [Nie87]. We have set the value of the free parameter for multiquadrics and inverse multiquadrics interpolants for Franke's test functions to be the one reported by Foley et al. [Fol94], which is nearly optimal for a slightly different distribution of data. For each of these functions, the interpolants can be constructed by sampling the analytic functions for different data distributions [Nie87]. Due to limited space, in this paper all the figures correspond to interpolants constructed by sampling Franke's first analytic function (that contains two hills, a valley and a saddle), on a 10×10 grid. Geometric uncertainty in these figures is computed as the difference between the geometric quantity of the interpolants.

In order to accommodate 8 color figures in one page, we have provided very short captions for the figures underneath. Here we describe each figure in greater detail. Except for Figure 3, which compares the C^2 bicubic B-spline interpolant with the bilinear interpolant, all other figures compare the multiquadric (MQ) with the thin plate spline interpolant. Figure 1 displays the volume filling glyph between the two interpolants. Figure 2 shows swept probes along a selected triangle for the MQ interpolant with probes and pseudo-coloring mapped to the difference between two interpolants. Figure 3 presents the displacement mapping between the two interpolants. Figure 4 displays a wireframe overlay of an isophote corresponding to the two interpolants with the displacement glyphs reemphasizing the difference between the two isophotes. Figure 5 displays difference of Gaussian curvatures of the two interpolants with pseudo-coloring mapped to the difference between the two interpolants. Figure 6 presents the mean curvature of the MQ interpolant with a dense display of crosshair glyphs indicating uncertainty in the mean and Gaussian curvatures. Figure 7 uses the transparency technique to display the differences between the two interpolants where the triangular glyphs represent uncertainty in the normals and the cross-hair glyphs represent uncertainty in the mean and Gaussian curvatures. Finally, Figure 8 displays the spot texture mapping between the two interpolants with greater spots in regions of higher uncertainty. The triangular strips indicate the uncertainty in the normals above a certain threshold.

4.3 Discussion

We now discuss the results of our experimentation with visualizing geometric uncertainty. The key observation is that a static visualization system is too highly constrained to be of much value in a practical situation. The key to a successful system is providing flexibility in creating visualizations by possible combinations of (i) visualization techniques, (ii) geometric uncertainty parameters, and (iii) visual parameters. This flexibility was heavily utilized in creating examples of visualizations presented in this paper and in conducting the experiments for probing the quality of surface interpolants. Examples and advantages of flexibility in choosing visual parameters are described in Section 3.2. Here we focus on analyzing the advantages and disadvantages of different techniques for visualizing geometric uncertainty.

Glyphs: We have found both the displacement glyphs and volume filling glyphs to be one of the most useful and precise techniques for comparing surfaces visually. Displacement glyphs give a very good idea of absolute differences between surfaces. They also provide the information as to where these differences are located as well as the relative positions of the two surfaces. Volume filling glyphs are very useful in providing a good sense of the error by filling the total volume enclosed between the two surfaces. Even if the absolute differences are rather small, this method can be made very effective by scaling the glyphs, by choosing different glyph shapes, by adjusting the spacing between glyphs and by zooming into the areas of interest. For example, spheres are better than boxes for small differences but worse for large differences because they tend to bulge out.

Texture Mapping: Displacement mapping, bump mapping and spot mapping provide relatively easy to view information about the regions where the two surfaces disagree. Although these methods seem to do a crude job of providing precise quantitative information, they are very effective both as additional cues and in having a clutter-free presentation even after adding more information about an additional geometric feature.

Transparency: Transparency uses much less data-ink to portray the same information and is very helpful in providing clutter-free presentation. This technique is also useful due to its see-through mechanism. However, this method does not provide a precise idea of absolute differences between the two quantities.

Difference Surface: This method is very effective in assessing the absolute difference between two quantities. By scaling, this method can also bring out regions of high relative differences. The location of these differences can also be grasped very easily relative to the domain, but not with respect to the range.

Overlays: Overlays provide satisfactory information about the relative placement of two surfaces or the two geometric quantities. However they are rather difficult to view due to intersections between two surfaces.

Pseudo-color: Pseudo-coloring technique is effective in bringing out the regions of high relative differences. However it is difficult to gain good understanding of the absolute value of the differences using this method.

Animation: We found it rather difficult to get much useful information from a simple animation between two surfaces. However when combined with animated probes that expand in proportion to differences between surfaces along prescribed curves over which they move, they become an effective method for detailed information in regions of interest.

Side-by-side comparison: This method is effective in revealing large structural differences only when they exist. However the eye cannot detect many subtle and even intermediate scale differences particularly when the differences are shifts of similar features.

5 Conclusions

In this work, we have described several techniques of visualizing geometric uncertainty of surfaces interactively. The user can create a wide variety of visualizations by choosing appropriate combinations of visualization techniques and geometric features of interest. The user is also able to perform interactive queries, select subregions of interest and map a variety of visual parameters in order to create useful and effective graphics. The system was applied to probe the geometry of surface interpolants.

Acknowledgments

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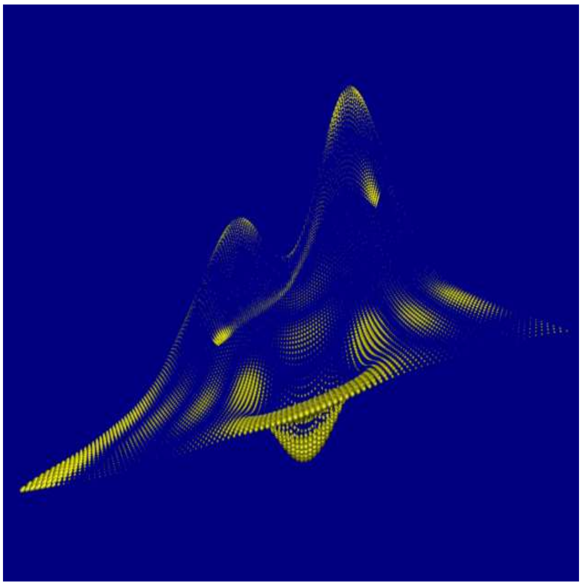

Figure 1: Volume filling glyphs

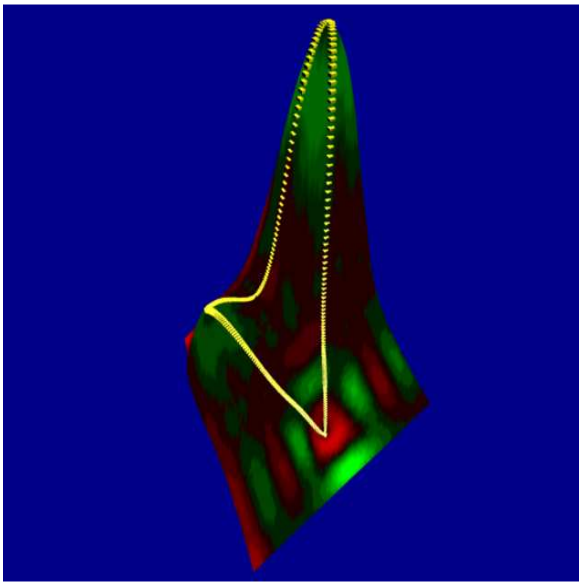

Figure 2: Swept probes

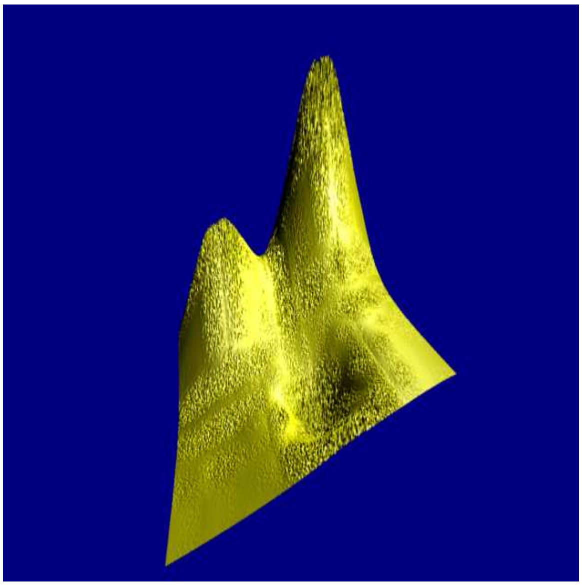

Figure 3: Displacement mapping

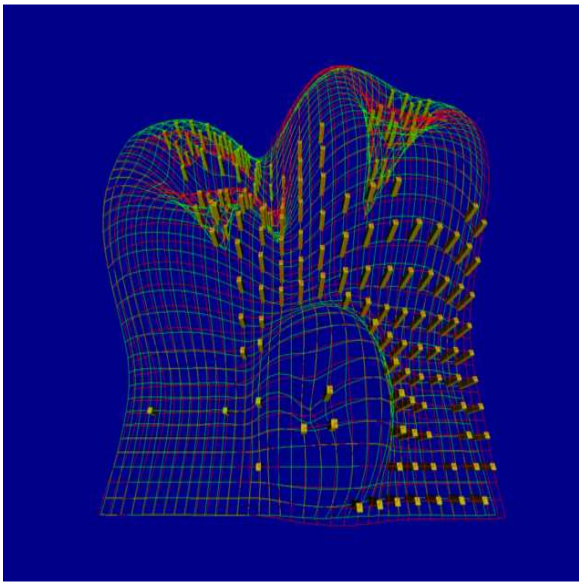

Figure 4: Displacement glyphs

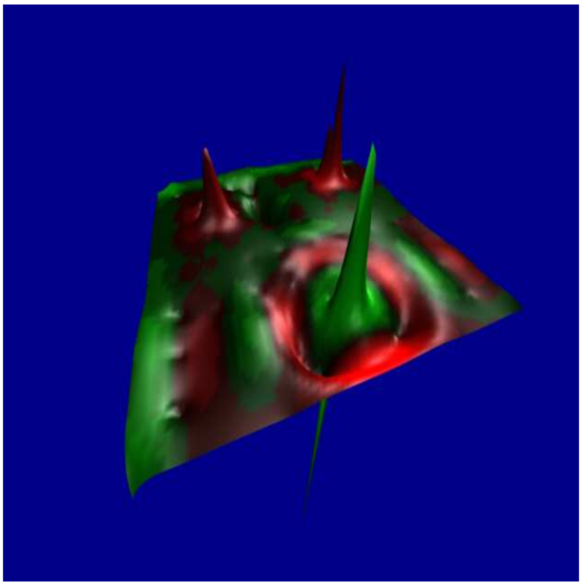

Figure 5: Pseudo-colored difference

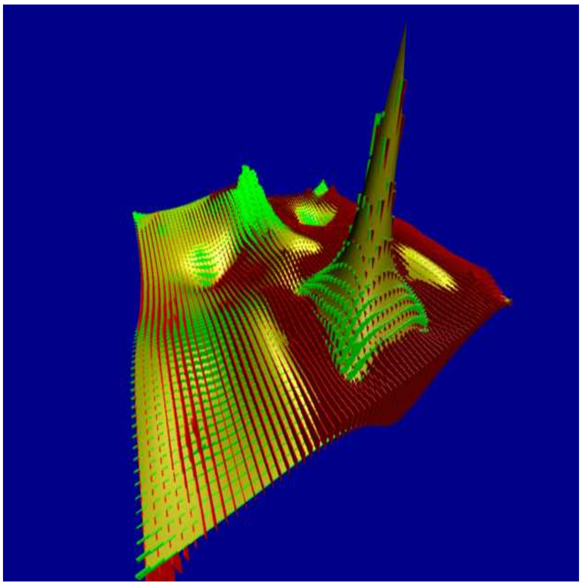

Figure 6: Cross-Hair glyphs

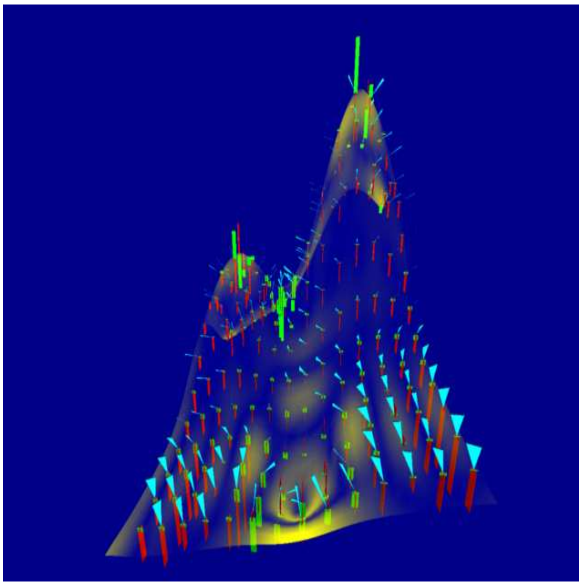

Figure 7: Transparency with glyphs

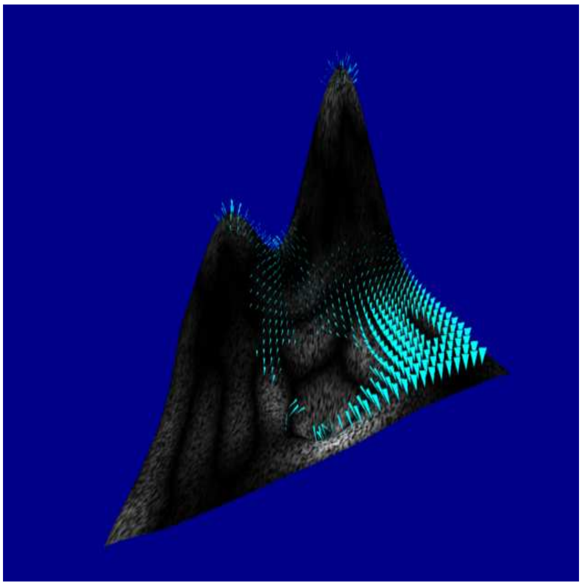

Figure 8: Spot texture with triangular glyphs