Visualizing Uncertainty in Natural Hazards

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Good visualizations are designed to answer a particular question or the needs of a particular task. They are created as comprehensively as possible taking into account numerous factors such as the application domain, established conventions in the community, nature of the question or task, technical constraints such as interactivity or large data sets, presentation concerns such as visual clutter and complexity, and the quality of the data. Care must also be taken to ensure that any artifacts are not inadvertently introduced into the visualization. This paper provides an overview of current visualization practices and techniques that incorporate data uncertainty in the presentations. Emphasis is placed on geospatial data sets. The paper also describes some of the challenges and research directions in uncertainty visualization research.

1 INTRODUCTION

"Presenting data without error rate is misleading". This is a quote from the OJ Simpson defense team regarding the presentation of DNA evidence without valid error rate statistics. Taken more generally, this practice is a prevalent shortcoming in the scientific and information visualization communities where data are visualized without any indication of their associated uncertainties. While it is widely acknowledged that incorporating auxiliary information about data, i.e. data quality or uncertainty, is important, the relative amount of work in this area is small. On the other hand, developments by the geographic, cartographic, and GIS communities in this regard is much more concerted. Some of the early efforts were spear headed by the participating members of the National Center for Geographic Information and Analysis (NCGIA) initiatives [5, 13], where different methods of displaying and animating data with uncertainty were proposed. An excellent summary of this body of work can be found in [32]. Combining these works with those from the information and scientific visualization communities, a typology for uncertainty visualization was presented which tries to map data, uncertainty, and tasks with the appropriate visual presentation [40]. Specifically, the typology for uncertainty visualization would give the user some guidance about the visual representations for the different types of uncertainty.

There are some key differences in the approaches between the first (geographers, cartographers) and the second (information and scientific visualization researchers). On one hand, the first group focuses on identifying and characterizing the type, nature, source, and characteristics of uncertainty; as well as map based uncertainty visualizations. On the other hand, the second group focus on the task of visually mapping the different facets of uncertainty, and extending the techniques to higher dimensional data sets as well. Clearly, we need to bridge this gap in order to provide an end-to-end solution to the users.

As prefaced in the abstract, good visualization need to target the results to the needs of the users. This means that not only do we need to identify who the users are, we also need to identify the particular task they're trying to do with the given data at that particular moment. Thus, the same data set can be presented in a number of ways – perhaps at various levels of detail, emphasizing or de-emphasizing different regions and features, and employing different visualization techniques to best present the message. However, there are also occasions were the users' goals are not known. In such situations, it is not uncommon to see visualization systems that try to provide interactive exploration of the data sets, or a flexible framework for specifying and emphasizing different aspects of the data set. Frequently, the tradeoff for having a flexible system is that the users need to specify more parameters before they can get their visualizations. Likewise, interactivity usually comes at a cost, both in terms of increased computational resources, but also in the quality of the renderings (e.g. tradeoff in quality versus speed [30]). Hence, the exploratory process needs to be followed by a refinement stage where feature extraction may be encoded as the users get a better grasp and can better define what the important features are, and the visualizations and user interfaces are streamlined so that the desired visualizations can be obtained with minimal effort. The description above is also reflective of the multidisciplinary nature of the visualization researcher in combining engineering, science, and art. Engineering in that the visualizations are problem driven with users trying to understand or look for features in their data sets and the visualization researchers specifying the best practice approach. Science in that the visualization researcher also need to draw upon various established fields such as perceptual and cognitive psychology, mathematical and physical analyses, etc. Visualization is also an art in that the results need to be tailored to the particular task, needs, and occasion.

What we just described is true whether uncertainty is taken into account or not. Uncertainty certainly does not simplify matters. It adds to the computational task of handling and presenting them, but also to the cognitive task of the users to understand them. This paper focuses on visualizing uncertainty in data sets, particularly those found in geospatial applications.

2 FROM CONCEPTS TO REPRESENTATIONS

Uncertainty is a multi-faceted concept and has include such concepts as imprecision, imperfect knowledge, inaccuracy, inconsistency, missing information, noise, ambiguity, lack of reliability, etc. Its many definitions are quite rich and reflect different properties of uncertainty, but at the same time provide no clear consensus or universally preferred meaning [34]. This has resulted in different ways of quantifying uncertainty [24], and included such measures as statistical variations or spread, minimum-maximum range values, data quality or reliability, likelihood and probabilistic estimates, etc. Likewise, numerical simulations involving uncertainties [25] and propagation and reasoning in the presence of uncertainty

[35] provide alternative means of manipulating the different flavors of uncertainty.

In this section, we quickly review the concepts of uncertainty, how they are represented numerically, and how we can go from there towards visualizing them. Note that there are much more comprehensive papers that describe the different concepts and facets of uncertainty such as those by Klir and Wierman [24] and Thomson et al. [40]. It is not the purpose of this paper to expand on those different concepts, but rather to focus on how the different concepts of uncertainty are ultimately represented numerically. The numerical representation is an important step in the visualization process as ultimately, these numerical representations need to be mapped to visual parameters.

In a typical visualization pipeline, we see a data acquisition stage where data are collected from measurements, field observations, and numerical models; these data then undergo a transformation stage where measurements may undergo unit conversions, new variables are derived from available ones, data may be refined or summarized, or features may be extracted from them; the results of the data transformation stage are then fed to the actual visualization step where the derived quantities are finally mapped to visual parameters. This pipeline is illustrated in Figure 1. We note that uncertainty can be introduced at any stage in the pipeline, including the final stage where the user may also misinterpret, or misuse the resulting visualization.

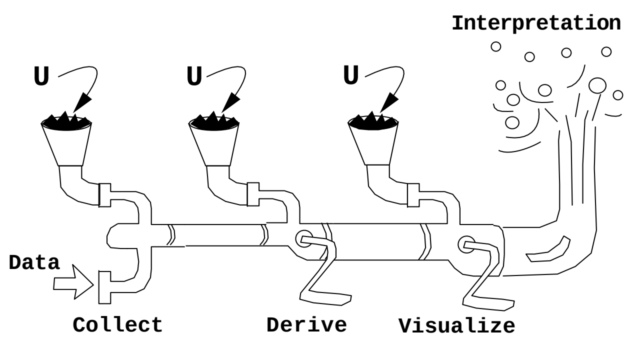

Figure 1: This visualization pipeline shows the introduction of uncertainty (U) from models and measurements, uncertainty from transformation processes, and uncertainty from the visualization process itself.

Uncertainty in acquisition:

Data are collected through observations and measurements as well as numerical simulations. In both cases, uncertainty can be introduced in various forms such as miscalibration or drifts in instruments, bias, noisy or missing data, and over simplification of mathematical models. These can be characterized as statistical variations [9]. With instruments, there is an experimental variability whether the measurements are taken by a machine or by a scientist. The more times the measurement is taken, the more confident the measurement. But there will be a statistical variation in these measurements. In numerical modeling, the model and its parameters have been decided by a domain specialist, and are inherently a simplification (e.g. linearization of a nonlinear system) of the system being modeled. Aside from model simplification and sensitivity of these models to input parameters, numerical calculations performed on these models also introduce errors due to the choice of integration algorithms and the limited precision of the computing machinery. Likewise, there is variability in human observations both in terms of difference in perception among individuals and also to slight differences when asked to perform a task repeatedly.

Uncertainty in transformation:

Often times, raw data are not rendered directly but are first subjected to further transformations with or without the knowledge of the person doing the visualization task. These data transformation operations may be as simple as conversion from one unit of measure to another, or may involve some algorithm to fuse one or more types of data together to derive a new data type. Data transformation operations may occur as early as the data acquisition stage or later in the visualization stage. Likewise, data may be rescaled, resampled, quantized, etc. either prior to, or as part of, the visualization stage. The key point is that these transformations alter the data from its original form, and have the potential of introducing some form of systematic uncertainty.

Uncertainty in visualization:

What is also interesting is that uncertainty is also introduced in the visualization stage itself. For instance, in global illumination of 3D scenes, radiosity algorithms use approximations for calculating form factors. Similarly, there are different approaches to direct volume rendering of 3D data sets [23] resulting in discernable differences in renderings of the same data set.

Uncertainty introduced in the visualization process is not limited to radiosity and volume rendering, but are also present in more routine operations. For example, the use of interpolation is quite prevalent in taking slices through data sets, in contouring, as well as isosurface algorithms [28, 43]. Similarly, in flow visualization methods, different integration methods, step sizes, orders, and seeding strategies lead to noticeably different flow visualization results. Effects of uncertainty are more pronounced in the vicinity of or on critical points in the flow field. These differences may at times result in drastically different flow visualizations.

Animation allows visualization to include an additional parameter, usually time. Again, there are several opportunities for uncertainty to be introduced. The process of in-betweening to fill in frames between key frames is analogous to surface interpolation, and while there is no single preferred method, there are many methods available, and all of them will result in some slight variations.

Hopefully, this short list of potential pitfalls illustrates the numerous ways in which uncertainty can be introduced into the visualization pipeline. Users should also be wary of blindly using visualization methods without fully understanding the limitations and assumptions of each method.

Comparing these different sources of uncertainty with the different concepts of uncertainty, one can also see that while most of the uncertainty may be introduced upstream during the data acquisition stage (e.g. missing information, noise, imprecision, inaccuracy), other forms for uncertainties may also be introduced in later stages of the pipeline (e.g. inconsistency in data handling procedures, lack of precision in numerical calculations, smoothing, fuzziness or vagueness from filtering operations, loss of information from sub-sampling, etc.).

What is critical for the uncertainty visualization task is to know how the uncertainty is numerically represented. That is, is uncertainty represented by a single scalar value that represents e.g. standard deviation or data quality? Or is it represented by a pair of scalar values that represents e.g. a minimum-maximum pair? Or is it represented by a whole range of values from measurements or simulations, or possibly by a probability density function? The different ways in which uncertainty is represented, coupled with whether this representation is available at individual spatial locations dictates to a large degree how uncertainty visualization can proceed.

Combined uncertainty and data representation:

We now describe a *multi-value* data type as a means of representing both data and its uncertainty. Note that this representation is not a cure all. In fact, it would not be able to represent missing data, for example. A multi-value is a data type which is simply a collection of n values about a single variable at a location p and time t: $M = [v_i]$, where $i = 1..n$ (see Figure 2). It is useful for representing uncertainty as multiple estimates such as those found in situations where there is variability in the measurement process or the physical phenomenon itself. A simple example is the measurements of temperature at a particular location using a variety of devices. The measurement from a single device is a single-valued data – which would also be a special case of a multi-value where $n = 1$. The collection of temperature values from all devices is a multi-valued temperature data for that location and time. Likewise, ocean and weather ensemble forecasts are generated from multiple runs of a set of models or parameters, and have multiple values for each physical field e.g. pressure, wind, current, temperature, at every spatial location and forecast period. Multi-valued data sets offer a richer representation of the variable nature of some data and can be used to represent uncertainty as well.

"Multi-valuedness" is a concept orthogonal to multidimensional or multivariate data. That is, a data set can be multidimensional, multivariate *and* multi-valued. Multidimensional data refers to the spatial dimensionality e.g. 0D, 1D, 2D, 3D, of the data, and often also includes time as another dimension. Multivariate data, on the other hand, refers to the different variables represented at each location. These variables are usually scalars e.g. temperature, but may also be vectors such as ocean currents, or tensors such as velocity gradients, and so on. Of course, a data set can be both multivariate and multidimensional: these properties are truly orthogonal. For example, a weather forecast may be 3D, time-varying and contain information about temperature, humidity, and pressure at each location. In practice, such data may be stored in a 5D array: three for space, one for time, and the last one for the different variables. A notable visualization system that carries this name is Vis5D [17]. Note that in this context, the term "dimension" includes the spatial, temporal and multivariate properties of the data. Multi-valued data adds an extra "dimension" in that we also need to represent the collection of values for each variable at each location and time.

Multidimensional, multivariate and multi-valued data are clearly different, as illustrated in Figure 2. One can have scalar multi-valued data where each member of the collection is a scalar (last row, first column), or one can have multivariate multi-valued data where each member of the collection is a multivariate vector (last row, last column). Clearly, the multi-valued data representation offers a much richer data representation compared to the single-value representation in use today. In the next section, we highlight the uncertainty visualization techniques leading up to multi-valued representations, and different approaches to visualizing data

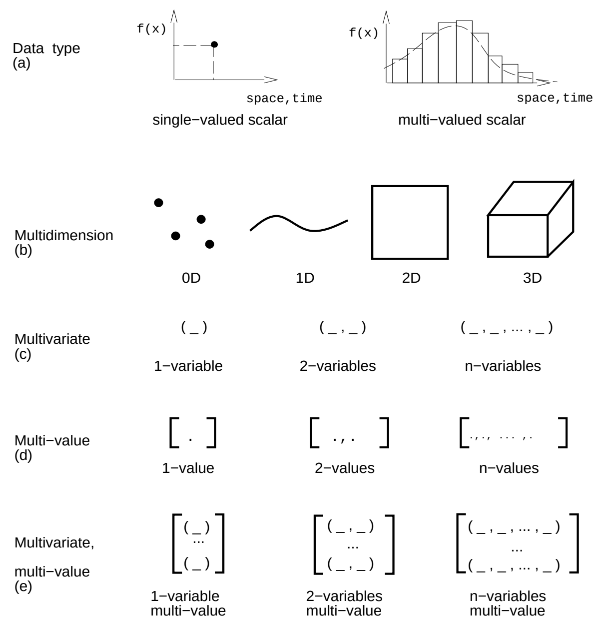

Figure 2: Multidimensionality (space and time), multivariateness, and multi-valuedness are orthogonal data properties. (a) illustrates the classic visualization of a scalar data, from a single value to a histogram for the multi-values. The next three rows illustrate the multidimension, multivariate and multi-valued properties. (b) illustrates the different spatial dimensions, while (c) illustrates the number of variables in a multivariate data. (d) shows the number of values in a single-variable multi-valued data. To make the representation explicit, the multi-values are surrounded by []. (e) illustrates a multi-valued data set with an increasing number of variables. Note that the graphical representations from (1c-1e) are for a *single* position in time and space. A multidimensional multivariate multi-valued data (not illustrated, this is a research challenge) would be represented as in the last row, last column, but replicated at every spatial location.

with such representations.

3 UNCERTAINTY VISUALIZATION

From the visualization point of view, we need to map data, including uncertainty, to visual parameters such as color, texture, transparency, geometry, glyphs, animation, etc. To make effective visualizations, we need to also ensure that these visualizations are designed to answer the particular questions or needs of the user. In this section, we look at the state of the art in uncertainty visualization. The latter goal of creating effective visualizations is a bit more difficult to generalize amongst the various types of user tasks without a more formal study. Instead, we will look at examples of how uncertainty visualizations can be used in visualizing natural hazards in the next section.

In the following discussion of uncertainty visualization, we organize the techniques according to the manner that uncertainty is represented.

3.1 Scalar Uncertainty

Majority of the uncertainty visualization research so far focus on uncertainty that are represented as single scalar values. To illustrate this point, let us look at a number of examples from the geographic/cartographic communities as well as the information/scientific visualization communities. Information about the driving application behind the visualizations is provided as needed. Additional details about the visualizations are available through the referenced works.

The idea of using blurriness, lack of crispness, or fuzziness to indicate degree of uncertainty is probably the most intuitive and widely used technique (see Figure 3). This method is probably best for data over a 2D spatial domain. Applying this technique to 3D (see left image of Figure 5) creates some ambiguity arising from the selected viewpoint. Furthermore, this technique also has the drawback of not being able to depict slowly changing degrees of uncertainty i.e. it has relatively poor resolution in terms of distinguishing amounts of uncertainty.

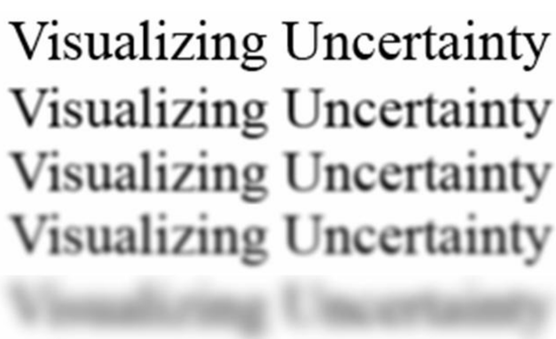

Figure 3: Uncertainty mapped to blurriness. Illustration of courtesy of Ben Schneiderman.

The effects of blurriness can be achieved by other means aside from a low pass filtering operation. For example, uncertainty can be mapped to transparency (see Figure 4) or noise (see right image of Figure 5). Comparing these two examples, the point based approach seem to have the advantage over the transparency approach particularly for 3D scenes where occlusion can be an issue in the interpretation of the amount of transparency.

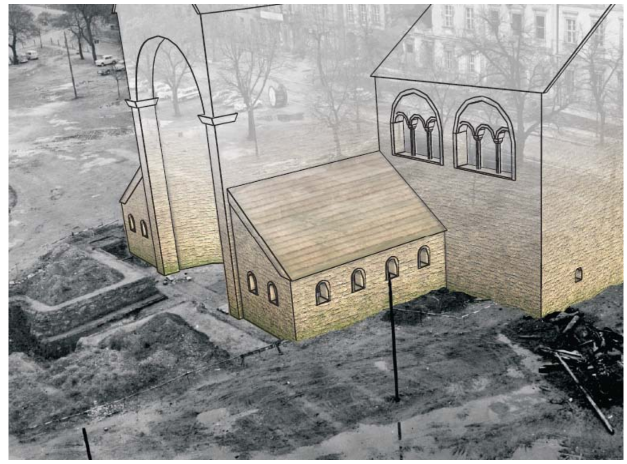

Figure 4: Uncertainty mapped to transparency. Remnants from architectural excavations reveal the locations of building foundations, coupled with information about the architectural style of the era, allowed the researchers to create a virtual reconstruction of the buildings. More opaque regions are more certain. Courtesy of Strothotte [39].

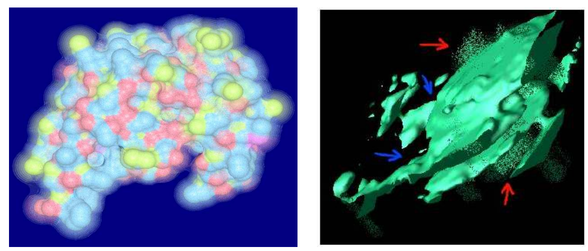

Figure 5: On the left, positional uncertainty is mapped to fuzzy surface. The data captures the atomic thermal motion as well as uncertainty of molecules using a Gaussian distribution [26]. On the right, positional uncertainty is shown using point clouds. The data is from a CAT scan of human kidneys with tumor formations. Arrows indicate positions of tumors. Red arrows are places where the uncertainty is higher [15].

A more drastic approach is to map uncertainty directly to a color map. The color map can either be continuous or discrete (see Figure 6). While this approach has the distinct advantage of allowing the user to easily gauge the degree of uncertainty, and can be used directly on a 3D scene, it uses one of the key visual parameters for mapping data values – color. Hence, if data plus the uncertainty about the data need to be simultaneously visualized, data would have to be visually mapped to a less prominent visual parameter.

Rather than mapping uncertainty to color, it can instead be mapped to saturation or value (using an HSV color model). Figure 7 illustrates this. In addition, the user can adjust a threshold below which the mapping simply grays out the uncertainty values. This mechanism allows the user to focus on the more “interesting” regions.

Drawing contour lines is one of the basic visualization techniques often applied to 2D data sets. Just as color can be used to indicate uncertainty, contour lines can also be used to indicate uncertainty (rather than data). An example of such use is in cartography where maps are generated with contour lines showing different amounts of angular or area distortions. Contour lines can be embellished so that

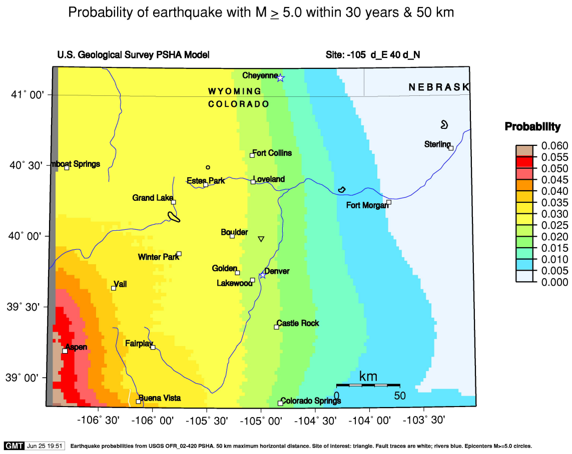

Figure 6: Probability that a magnitude 5.0 or greater earthquake will hit the Boulder area in the next 10 years is mapped to a discrete color scale. Image produced by the USGS Earthquake Probability Mapping at eqint.cr.usgs.gov/eq-men/html/neweqprob-06.html

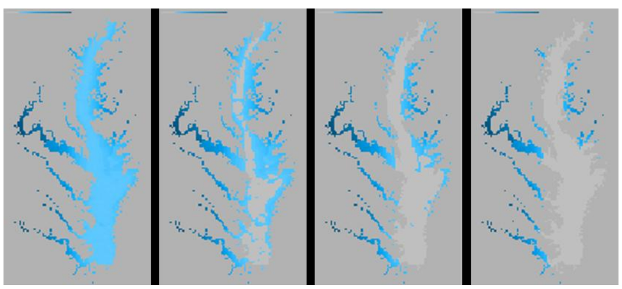

Figure 7: Left images show the 95% confidence interval where darker regions have higher values. Users can then progressively adjust the focus to only show those regions. [31]. The data is about the confidence levels of the dissolved inorganic nitrogen in the Chesapeake Bay.

it can depict the underlying data, while the embellishments depict the uncertainty about the data. Figures 8 and 9 illustrate three such embellishments: (a) varying the thickness of the contour lines, (b) varying the brightness of the contour lines, (c) varying the connectedness of the contour lines. Other variations such as using noise, color, texture or other visual parameters can also be applied to contour lines.

Another approach proposed by Cedilnik and Rheingans [8] is to overlay the domain with a grid, and apply modifications to the grid proportional to the amount of uncertainty (see Figure 10). Conceptually, this technique should be extendible to data sets covering 2D manifolds in 3D space as well. One caveat is that the grid (or whatever texture) can potentially obscure the underlying data as well.

The examples shown above are for scalar uncertainty values. Positional uncertainty using Euclidean distance, confidence level, and probability are all scalar terms. In the next example (see Figure 11), we look at a 2D vector field where the angular uncertainty is used to alter the traditional arrow glyph. In addition to angular uncertainty, magnitude uncertainty can also be encoded into these uncertainty glyphs. Care must be taken in designing and using uncertainty glyphs. For example, it is less confusing to map the velocity magnitude to the area of the glyph rather than the usual mapping to length of the arrow [45].

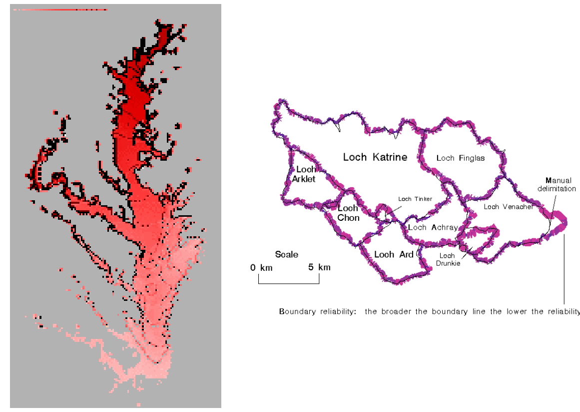

Figure 8: On the left, a contour is overlayed on top of the data map. The contours show uncertainty and grow thicker as the uncertainty rises [31]. The data set is the same one as in Figure 7. A similar idea is used on the right image where the reliability of watershed boundaries is inversely mapped to the width of the boundary lines [33].

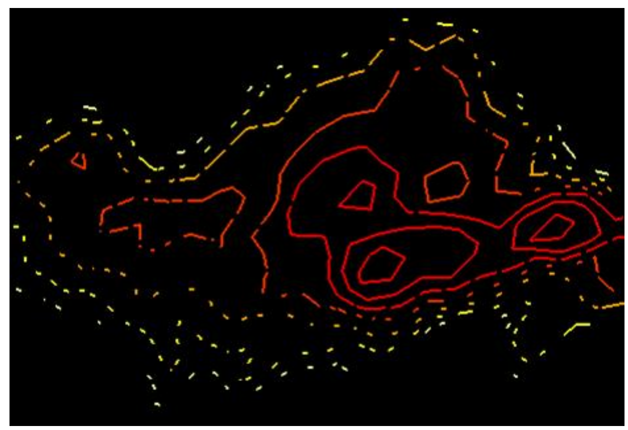

Figure 9: Yet another variation on how uncertainty can be depicted on contour lines. The contours show uncertainty and appear more broken as the uncertainty rises [11]. Color of the contour lines is available for mapping to another variable.

In summary, most of the uncertainty visualization research has focused on situations where the uncertainty is represented as a scalar term. Several techniques are now available for including scalar uncertainty in the visualization of scalar 2D and 3D data sets. In the last example, we have shown an example where uncertainty in a 2D vector field is visualized. Research is also ongoing in showing uncertainty in tensor fields such as those found in diffusion tensor imaging [4] and in geomechanics simulations. Omitted from the examples above are animation techniques where uncertainty is mapped to different animation parameters [12]. But the applications where such techniques have been applied usually have scalar uncertainty terms.

In the next two sections, we look at examples where the uncertainty is represented by more than a single number.

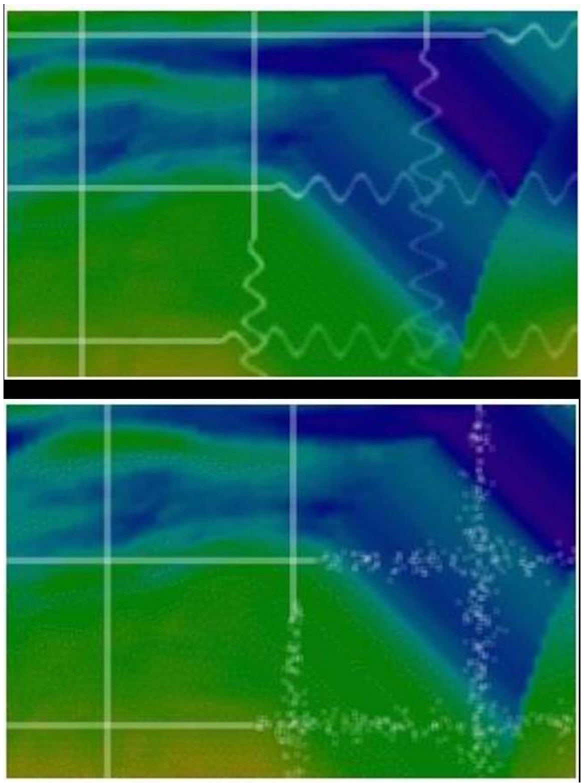

Figure 10: A grid is superimposed over the 2D domain. The data values are mapped to color, while the grid is modified according to uncertainty values. On top, it is mapped to amplitude, while on the bottom, it is mapped to noise [8]. The data set shows the total ozone measurements made by the Nimbus-7 satellite on September 16, 1979. Onboard sensors are subject to drop outs hence leading to uncertainty in portions of the data set.

3.2 Vector Uncertainty

Here, the term vector refers to a feature vector, or a set of numbers that characterize the uncertainty at a particular location. A common example is to use a pair of numbers i.e. minimum-maximum value pairs to represent the bounds of an interval where the actual data value may lie. This is illustrated in Figure 12 where the min-max pairs define the extent of the bar glyphs that are drawn over the mesh surface. This technique works best on 1D data sets. Extending it to 2D data sets such as the one shown in Figure 12 can potentially obscure how much above or below the surface the min-max pair is, specially if the surface is rendered e.g. transparently. Drawing bars to represent the pairs does not extend very well into 3D volumetric data sets.

In situations where multiple statistics, or a vector of features is available, then multivariate visualization techniques may be applicable. An example of this is to map (up to 3) values to an HSV color model [29]. Figure 13 shows such as example. Since the mapping is to a color model, the technique can be applied to 2D and 3D data sets. However, where surface shading is used, it may cause some ambiguity with the value mapping.

We do not find as many examples where uncertainty is represented as a feature vector. But in cases where they appear, glyphs, color mappings, and other multivariate visualization techniques are applicable.

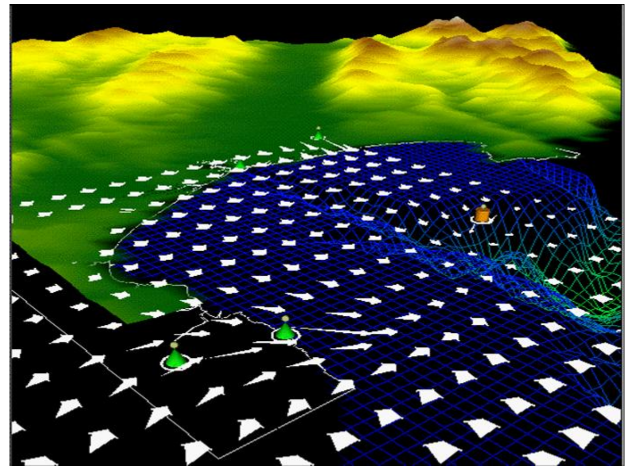

Figure 11: Angular uncertainty represented as angular spread is mapped to the width of the arrow head [45]. Data is derived from correlating radial vectors from two different sensor locations. The derived 2D vector field has higher accuracy closer to the sensor locations and drop off with distance as well as the angular difference between the two sensor locations. That is, if the radial components from the two sensors have less angular difference, the derived vector also has less accuracy. [45] also describes methods for showing uncertainty in velocity magnitudes in the derived vector field.

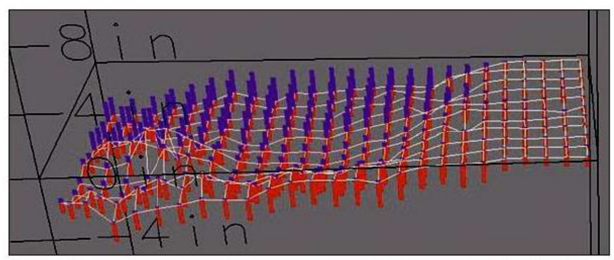

Figure 12: Minimum and maximum pairs rendered as bars below and above the wireframe surface respectively. The data indicates future water balance change predictions [10].

3.3 Multi-value Uncertainty

When data comes in the form of a multi-value at every point, we have information about the data values as well as the ability to derive additional information about the collection. For example, given a single multi-value datum, we can calculate some parametric statistics about that datum such as mean, standard deviation, minimum, maximum, inter-quartiles, etc. These data descriptors can then be visualized with, for example, a box plot. Alternatively, the data could be binned and rendered as a histogram. This is illustrated in Figure 14. An obvious limitation is that this method, as well as the box plot method, does not scale well with spatial resolution; neither does it scale with higher dimensional data.

An alternative approach is to display the properties of the multi-value on a separate location or geometry (see Figure 15). Examining the left and right walls, one can observe that the distribution of values on the left portion of the left wall and the right portion of the right wall (which corresponds to points in the vicinity of the corner in the foreground) have a bimodal characteristic. Looking at the left wall also reveals that there is a shift in the mean value about halfway

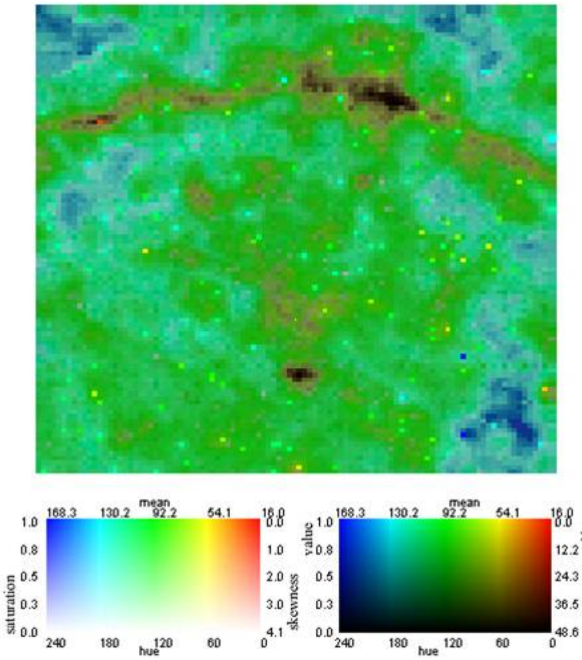

Figure 13: HSV color mapping of three variables. Hue is mapped to the mean of the data values, saturation is mapped skewness, and value is mapped to standard deviation. Dark regions represent areas of high standard deviation or uncertainty. Brighter regions represent places where the data values are skewed to the left – in this particular data set, those regions correspond to places where ground truth measurements were taken [29]. The data set is from conditional simulations of "forest coverage" over a small region in Netherlands [21]. The visualization highlights locations with ground truth readings, areas of higher uncertainty (standard deviation), as well as mean characteristics of the conditional simulations at each spatial location.

across, with a corresponding increase in spread of values and reduction in peak height. While such visualizations can offer more insight into the properties of this 2D multi-value scalar field, there are 2 drawbacks: (a) It does not show the entire 2D multi-value field at once. Instead the user has to interactively explore the space by selecting different transects or slices through the data space. (b) This approach does not scale with spatial dimensionality and therefore does not work well with 3D multi-value scalar fields.

Using multivariate visualization or the GIS layered thematic approach has similar limitations as illustrated in Figure 16. This approach also suffers from inability to scale beyond 2D data sets.

A more general approach to dealing with multi-value data sets is an operator based approach [29]. Here, mathematically and procedurally defined operators work directly with multi-values as a data type. Using this approach, traditional visualization techniques such as contour lines, isosurfaces, streamlines, etc. can be modified to work directly with multi-values. Figures 17 to 19 illustrate this.

Multi-values are much richer representations of data and their associated uncertainty. Visualizing multidimensional, multivariate, multi-values is quite challenging, although the operator based approach does provide a means for attacking this challenge. Because of the large degree of freedom on how operators are defined, the subsequent interpretation of the visualizations must be done with care. Users must also

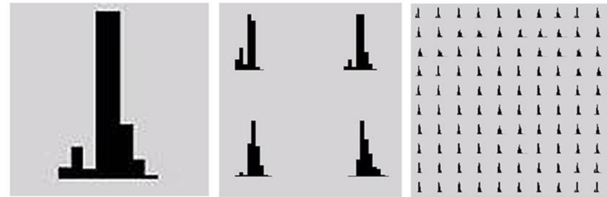

Figure 14: Multi-value rendered as a histogram. While this approach works for individual data points or low spatial resolution data, this approach does not scale well with spatial resolution and dimension.

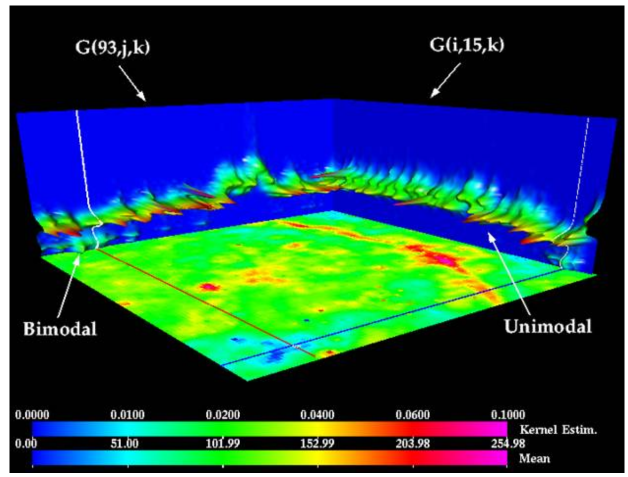

Figure 15: The multi-values corresponding to those found on the right line are displayed as a colored surface on the left wall, while those found on the left line are displayed on the right wall. The height of the walls corresponds to the range of values in each multi-value, while the color on the walls represents the frequency of values. The color on the flat surface corresponds to the mean of the multi-value at each location. The data is the same as in Figure 13. This visualization shows more detail about the characteristics of the multi-value at each location e.g. modality, shifts in mean, etc; however, it is done at the expense of looking at only part of the data at a time rather than at the entire 2D field simultaneously.

be educated on how to interpret such visualizations.

4 TASK ORIENTED VISUAL MAPPINGS

In the previous section, we looked at various ways in which uncertainty information can be mapped to visual parameters. In the case of multi-values, one of the key problem was how to pack as much information into the display.

In this section, we relax the criterion a bit and instead we first decide what subset or feature of the multi-value (or uncertainty) information needs to be presented. We should also preface this section that there has been a few notable work on the more theoretical aspects of effective visualization particularly from the point of view of perception. Bertin [6] described 8 variables (plane – 2 variables in a 2D domain, size, value, color, grain, orientation, shape) and identifies whether they are selective, associative, ordered, or quantitative. Tufte [41] advanced the idea of graphical excellence where a visualization gives the viewer the greatest number of ideas in the shortest time with the least ink in the smallest space. Ware [44] brings the physiological and cognitive psychological views such as pre-attentive process-

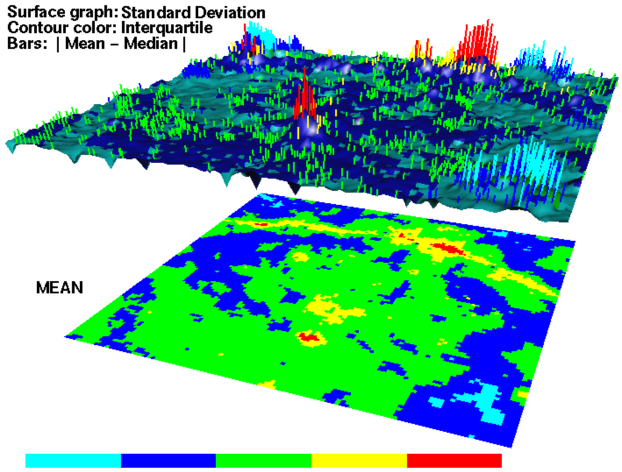

Figure 16: The bottom plane is the mean field colored from non-forest (cyan) to forest (red). The upper plane is generated from three fields: the bumps on the surface is from the standard deviation field and colored by the interquartile range; the heights of the vertical bars are from the absolute value of the difference between the mean and median fields colored according to the mean field on the lower plane. Only difference values exceeding 3 are displayed as bars to reduce clutter. The data is the same as in Figure 13. This approach uses parametric statistics and assumes the multi-values can be adequately described by these statistics. However, for ill-behaved distributions, or higher dimensional data sets, this thematic layered approach is inadequate.

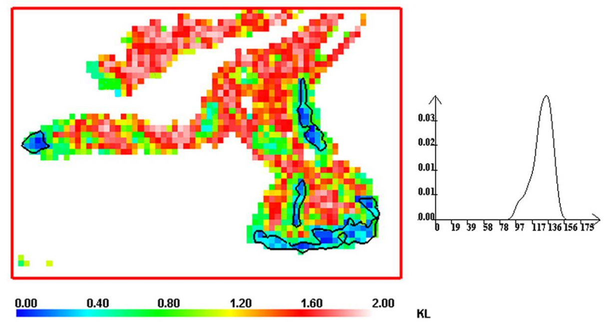

Figure 17: Contour lines showing where the multi-values in the 2D field are similar to the distribution shown on the right. The data is from multi-return LIDAR surveys of an Alaskan island [22]. Each location of this 2D multi-value field shows the tree heights within a 10 × 10 square meter stand of trees. The distribution of tree heights at each location sheds important information about the trees e.g. old growth stand, storm damage and regrowth, etc. Using this visualization, the scientist found all locations on the map where the tree heights correspond to those found in areas recovering from a weather disturbance event.

ing, gestalt laws, memory, eye movement patterns, etc. to bear on designing effective visualizations. Zuk and Carpendale [46] used components from Bertin, Tufte, and Ware to analyze different uncertainty visualization methods. Another measure of effective visualization can be taken from the viewpoint of how well the visualization aided a particular task. In this regard, the key issue that Thomson et al. [40] are addressing is how to map different types of uncertainties (e.g. accuracy/error, precision, consistency, lineage, currency/timing, credibility, subjectivity, interrelatedness) to visual metaphors. As an initial step, they have identified the underlying models for the different types of uncertainties and are in the process of finding the appropriate visual metaphors for the different models.

While it is important to be aware of perceptual issues in designing the visualizations, it is equally important to be aware of the cognitive issues in terms of how viewers internalize and understand the visualizations and how the visualizations influence their decisions and actions. Hence, we include a few examples from cognitive psychology research that the visualization community can learn from in terms of right and wrong ways of presenting information. First of all, decision makers are able to process and use only a limited number of variables particularly when under time pressure. Therefore, it is foremost that presentations are kept simple and reserved for the most critical information in their decision process. In a recent study, Peters et al. [36] found evidence that suggests a tradeoff between the completeness of data provided versus its comprehensibility – that in fact increasing the completeness of information can decrease comprehension and use of information in decisions. This suggests that information presentation should not overload the cognitive tasks, and selective pre-processing such as feature extraction, can alleviate the cognitive load by helping the users focus on the important aspects of the data. Secondly, the manner in which the information is presented can influence how decisions are made. For example, Slovic et al. [38] found that a risk factor of "20 out of every 100 persons similar to Mr. Jones are estimated to commit an act of violence" was perceived as being riskier than "persons similar to Mr. Jones are estimated to have a 20% chance of committing an act of violence". Slovic et al. [37] also conducted a study that suggests individuals will "image the numerator" and "neglect the denominator". Hence, information presented as "115 out of 10,000" and "2 out of 10,000" are more likely to be comprehended and used as opposed to the equivalent information presented as "23 out of 2,000" and "1 out of 5,000" respectively. These results suggest that visualization designers should also be careful in choosing the numeric scales and manner (e.g. frequentist) in which data is presented because the visualization can influence the decisions of the users in unintended ways. Another important finding is that we tend to conceptually simplify spatial entities [42]. For example, curves are often remembered as straighter as they actually are, angles of intersections are schematized to 90 degrees, and areas of regions are diminished in memory. This concept of the mind simplifying spatial entities has been used successfully in maps [1]. The visualization community can likewise take advantage of this trait in presenting complex information.

In the context of hazard communication, there are many stakeholders that need the information e.g. planners, emergency response teams, media, public, etc. Obviously, different stakeholders different needs and uses for such information. A "one-size-fits-all" approach in hazard visualization may therefore not be the right approach. As an alternative, one can identify classes of users, types of tasks, complexity of data they are dealing with, and then attempt to find a framework that matches one or more visualization methods that has been shown to effective from best practices. The success of this approach depends on how well each category of users, tasks, and data is representative of the particular instance at hand. That is, how good is the resolution within each category, and how well does each element represent members of that category. An initial coarse categorization

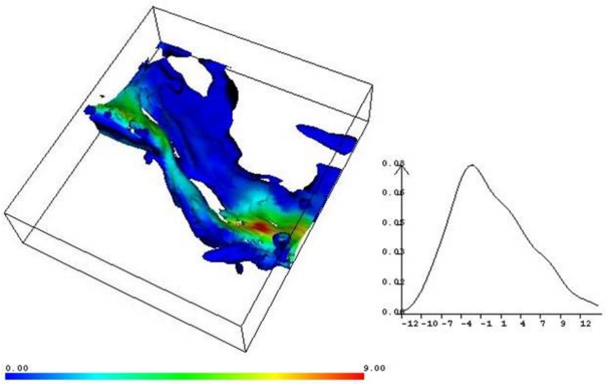

Figure 18: An isosurface using a reference temperature multi-value is shown on the right. The surface represents regions in the data where the multi-values are very similar to the reference multi-value. Color of the surface shows the standard deviation of the multi-value at each location. The data set is from an ocean circulation model of the Middle Atlantic Bight shelfbreak which is about 100 km wide and extends from Cape Hatteras to Canada. Both measurement data and ocean dynamics were combined to produce a 4D field that contains a time evolution of a 3D volume including variables such as temperature, salinity, and sound speed [27]. An ensemble of 600 Monte-Carlo forecasts of each field was produced.

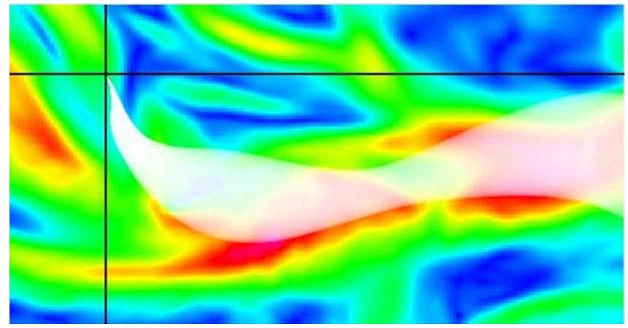

Figure 19: Streamline visualization of ensemble weather forecasts [29]. It is rendered with overlapping transparent circles. The effect is similar to using blurriness to depict uncertainty. This data set is courtesy of NOAA's operational forecasts (http://wwwt.emc.ncep.noaa.gov/mmb/SREF/SREF.html). The ensemble was created from two different models: ETA and RSM, with 5 different initial and boundary conditions each producing an ensemble or collection of 10 members at each location where the two models overlap. Unfortunately, the two models are not co-registered and have different projections and spatial resolutions. Thus, for the purpose of this paper, we just used the five member ensemble from the RSM model. The resolution of the RSM model is 185 $\times$ 129 and has 254 physical variables at each location. Velocity is available at every location in the model. However, only horizontal wind components are recorded and that is what is shown in this figure.

can be as follows.

Types of users:

1. *Scientists, Engineers, Doctors.* This group of users are experts and familiar with the data sets they are working with. They are usually looking for a known feature in the data set e.g. location and extents of weather fronts or shockwaves; maximal stress points in structural design; and existence, presence, size of tumors.

2. *Policy Makers, Decision Makers, Court Cases.* This group of users may not be very familiar with the data set, but need to get a high level understanding and the potential impacts provided by the data set.

3. *Operational Users.* This group of users have a fairly well defined set of visualization products that they need.

4. *Casual Users.* This group of users would generally have the lowest technical expertise about the data. They are similar to the second group above, but their use of the visualization may be for educational/informational purposes, or the decisions they make based on the visualizations do not have as large a consequence.

Types of tasks:

1. *Analysis.* The types of tasks that fall under this category may include feature extraction, identification, quantification, comparison, etc.

2. *Monitoring.* Usually, the task itself is passive, and is event-driven e.g. out of the ordinary trigger events. There is also some degree of assessment to gauge the severity of the event and to decide if an alert needs to be issued.

3. *Exploration. Data Mining.* Here, the user does not necessarily know what features to look for. They are looking for "interesting" aspects or perhaps hidden relationships in the data set. The tasks are therefore investigative by nature, and may even include "What-if" type questions.

4. *Persuasion. Communication.* The results or messages are known, and the user simply wants an effective way of conveying them.

Types of data:

1. *Data dimensionality.* This refers to the spatial and temporal dimensionality of the data. Related to this is mathematical manifold e.g. a 1D data could reside in 3D space as a curve.

2. *Data type.* This refers to whether the data is a scalar, vector, or higher order tensor quantity. Usually, visualization will focus on showing derived quantities from higher order tensors.

3. *Multivariate data.* This refers to how many separate variables are available at each physical location, or sampling/measurement event.

4. *Multi-value.* This is as described in the earlier section of this paper.

5. *Ordinal, Cardinal, Categorical.* This mainly refers to whether the data can be ordered or grouped.

Given the categories above, the next task is to create a framework where different visualization methods can be used to match the needs of a particular user, task, data combination. This framework would be applicable whether uncertainty is a concern or not. Thus, if uncertainty is the primary concern, then the data category corresponds to the underlying model classification in [40]. On the other hand, if data and uncertainty both need to be visualized, the previous section illustrated some of the available techniques. In the next section, we will examine a few specific cases of hazard visualization and how they may relate to this framework.

5 HAZARDS VISUALIZATION

Natural (and man made) hazards occur at different time scales. Environmental effects from pollution, logging practices, etc. take longer before the impacts are evident. On the other hand, fire, hurricane, flooding, earthquake, terrorist threats require a more urgent response. Different types of users will be interested in these hazards in different ways. The technical professionals would be interested from the point of view of modeling and prediction of these phenomena; the decision makers would be interested to know the likelihood of when, where, how significant or catastrophic the event might be; the operational users would like to know if there's sufficient basis to recommend an alert or emergency procedure; while the casual user may be more interested in safety of their loved ones, or perhaps evacuation routes from approaching hazards.

With regards to tasks, examples related to hazards visualization may include: forecast and track the trajectory of a hurricane, determine the likelihood of an earthquake occurring at some region within a certain time frame and certain magnitude, study the cause and effect relationships of rise in sea surface temperature to migratory patterns of birds, monitor computer server traffic patterns for denial of service threats, etc. We look at how visualization and uncertainty play a role in two applications.

5.1 Seismic Application.

Seismic activities can trigger a chain of events that require immediate action. Aside from the immediate damage from a strong earthquake, it can also cause subsequent damage in the form of fires and tsunamis. Liquefaction causes much destruction in earthquakes, and their characterization is subject to uncertainties in determining the relevant properties of natural soil mass [3]. The visualizations needed for seismic applications can vary from modeling/analyses of movements on fault lines, soil liquefaction and impact on surrounding structures, etc., to planning the best evacuation route in case of flooding, maps showing how to best deliver disaster relief services, etc.

Some of the visualizations available on the web include the following: (i) Figure 20 shows the 3D structure of underground and underwater faults, (ii) Figure 21 shows frames from an animation of seismic wave propagation towards the surface, (iii) Figure 22 shows a USGS hazard map.

As illustrated in the figures, earthquake visualization spans the range of ground motion estimation in 2D and 3D, and characterization of fault structures in 3D. What has been omitted is the large body of work aimed at preventing or mitigating disasters by designing structures to withstand severe ground movements. In this regard, the data type of interest are usually 2nd order symmetric tensors that represent stress and strain, as well as 4th order constitutive relationships [16] that describe how materials respond to stress over time. With the exception of Figure 6, the seismic related visualizations do not really portray uncertainty. And even with Figure 6, the visualization is straightforward in the sense that the data is 2D, the uncertainty is a scalar term, and no other underlying data need to be presented simultaneously. This type of visualization seems to be representative of those found in www.hazus.org

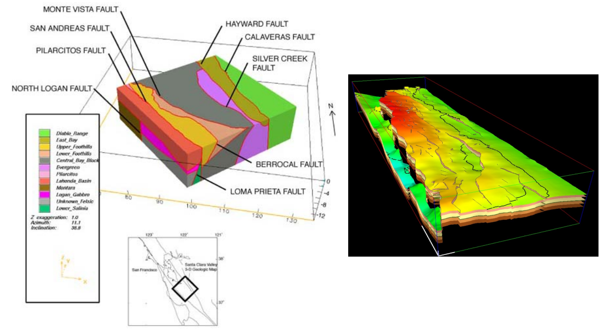

Figure 20: On the left, fault block diagram showing 9 of the 11 major faults in the Santa Clara Valley [18]. On the right, the physical structure of a North Sea oil field derived from 3D seismic data [2].

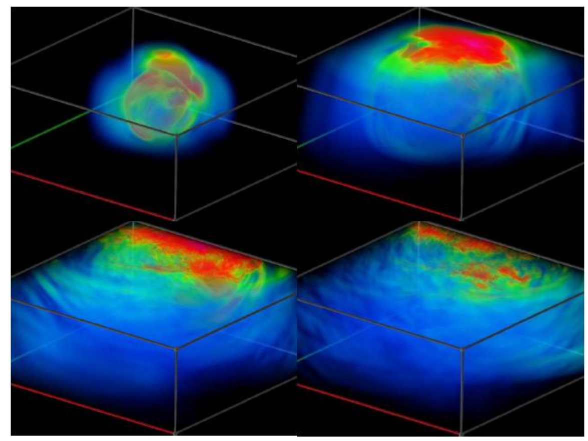

Figure 21: 4 direct volume rendered frames from an earthquake-induced ground motion simulation [30].

as well. Perhaps the reasons for the lack of uncertainty visualization in this field are because uncertainty is difficulty to quantify in a meaningful way (lack of data, lack of knowledge, etc.); the scientists and engineers are still grappling with how to visualize 3D second order tensors, not to mention 4th order tensors. Nevertheless, there is a need to focus on developing uncertainty visualization techniques in this area as probabilistic models are being employed to study elasto-plasticity [19] and the simulations are generating essentially multi-value at each location. In short, if we try to examine the needs of the different users within the seismic application related communities, we find that the current data acquisition (modeling and measurements), and particularly the visualization tools and techniques are still quite rudimentary. We obviously still have quite a few hurdles to overcome before meeting their needs.

5.2 Weather Application.

The spectacular force of nature is usually felt in severe weather disturbances like hurricanes. These hazards happen on a relatively periodic and predictable pattern, and with significant consequences to lives and property. Because the time scales is such that advance warnings can greatly save lives, there is much more research and advancement in this area.

Output from weather forecasting models, grouped to-

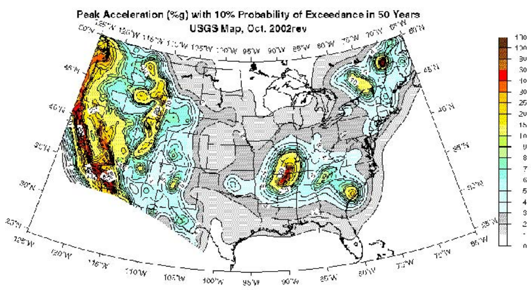

Figure 22: Hazard map showing regions with the same 10% probability of exceedance in 50 years, but with different degrees of ground motion [7]. Warmer regions do not necessarily indicate areas that are more likely to experience an earthquake, just that if it does happen it would be more severe.

gether into an ensemble forecast, coupled with in-situ and remote satellite measurements, are assimilated and used to refine nested models, to produce fairly accurate weather forecasts. However, the degree of accuracy may vary quite a bit depending on a number of factors such as the typical climate of the region, micro-climates, size and intensity of the disturbance, and how far out in time the forecast is for.

While accuracy of a weather forecast is broadly used these days, for example, weather report stating 30% chance of shower, or the projected path of Hurricane Katrina (see Figure 23), the accuracy, or alternatively, the uncertainty, is presented at a fairly coarse level. 30% chance of shower within which smaller geographic region, or narrower time window?; or in the case of Figure 23 – is the hurricane track equally probable within the white region, or is it higher in the middle of the path? How about the accuracy of the strength and estimated arrival time of the storm? Such information are important for decisions such as whether one should bring an umbrella, or to go to the beach, or to initiate evacuation procedures, etc.

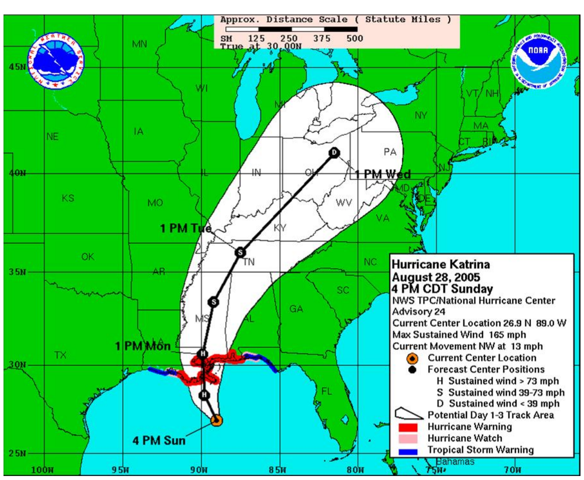

Figure 23: Projected path of Hurricane Katrina.

Uncertainty visualization for the weather application is in better shape compared to the seismic application. However, there are rooms for improvement. For example, the process of generating Figure 23 could be automated. One way to do this would be with multi-value streamline integration as shown in Figure 19. While winds from hurricane can cause a lot of damage, the associated flooding and landslides can just be as deadly. It would be beneficial to integrate the modeling of these events and couple them with topographic and levee system information.

6 CHALLENGES

Decisions can only be as good as the quality of information to work with. However, when dealing with real world data, and even with simulation data, data uncertainty is a fact of life. The presence of uncertainty should not only be acknowledged, but we also need to have a concerted effort to account for them, quantify, represent, track, and operate directly on them.

In our driving science applications related to natural hazards such as, geomechanics, oceanography, and weather forecasting, we need to deal with spatial dimensionality of 2 to 4 (space and time) and with highly multivariate interactions. Adding uncertainty, specially in the form of multi-values, can be quite challenging from the visualization point of view. Furthermore, multidimensional multivariate multi-valued data sets are inherently much larger and hence present a computational and informatics challenge in itself. Johnson and Sanderson [20] mentioned the development of new uncertainty visualization techniques as one of the key challenges. Dealing with multi-valued data certainly falls under this category. Griethe and Schumann [14] cites the lack of uncertainty visualization techniques for abstract data as another research challenge. This is certainly true as higher level abstractions need to be presented from these scientific based applications in order to facilitate the task of decision makers.

In summary, the theoretical and computational capabilities of current visualization techniques need to be extended. In particular, the research agenda should include:

1. A formal and theoretical framework for uncertainty visualization as proposed by [40] but one that is also cognizant of the users needs and tasks as well as the properties of the data that they are dealing with.

2. Research on uncertainty representation that captures its multi-faceted nature e.g. data are from multiple sources with varying degrees of reliability, etc. This includes visualization techniques that incorporate multi-values. In the earlier examples, we showed some visualization techniques that work with multi-values. However, there is a lot more research needed. For example, we still don't know how to do direct volume rendering on such data sets, nor do we know how to do critical point analyses on such data sets.

3. Research on data analyses techniques that take advantage of such representations. This may also include feature analyses and extraction as a means of condensing the important aspects from large data sets.

4. For the general area of seismic applications, we need to advance basic visualization techniques for 3D 2nd tensors fields, 4th order tensor fields, as well as probabilistic tensor fields.

5. A broader research into the perceptual and cognitive processes in how we digest and act upon visualizations, particularly those that contain uncertainty information.

Uncertainty is an integral part of the data that we look at on a daily basis. The tools that we develop should reflect the nature of such data, and allow us to explicitly see them and factor their influence in our decision making.

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